ECON 6310
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Practice Problems

Problem 1. Take the simple deterministic model for which $u(c_t) = ln(c_t)$, $k_{t+1} = k_t^\alpha - c_t$ where we set $A = 1$.

a. Find the first order condition for c_t.

b. Solve for the steady-state values for consumption and capital as a function of the parameters α and β..

c. Using Matlab's built in functions *syms.m* and *solve.m* find the steady state level of capital and consumption assigning values $\alpha = .33$ and $\beta = .95$ using the following commands:
syms c k
$tic; S = solve(k + c - k^\alpha, 1 - \beta\alpha k^{(\alpha-1)}); toc$
S=[S.c S.k];
S=eval(S)
You have to actually plug in the values for α and β into the expressions in the *solve* function and use the caret symbol to raise your variables to the powers determined by the specified values. Notice that we also multiplied each side of the FOC for consumption by steady state consumption. Verify that Matlab's answers correspond to the true values from your closed form solution.

Problem 2. Take the simple stochastic growth model for which $u(c_t) = ln(c_t)$, $k_{t+1} = \theta_t k_t^\alpha - c_t$ where we set $A = 1$.

a. Using the program *SolveLA.m*, solve the model with using the linear approximation with the same values for α and β as presented in Problem 1.[1]

b. Solve for the policy function for consumption in *levels*. You should be able to solve for consumption function as a linear function of the state variables so that:

$$c_t = B1 + B2k_t + B3\theta_t \tag{1}$$

[1]Recall the code default inputs should be:
$[Lxx, Lxz, Llx, Llz, Lux, Luz, Clevel] = SolveLA(Cu, Cxl, Cz, Dxl, Fxl, Du, Fu, Dz, Fz, .95, 1, 1)$

where $B1$, $B2$, and $B3$ take real values.

c. Given that we know the true policy for consumption takes the form $c_t = (1-\alpha\beta)k_t^{\alpha}\theta_t$, how well does the linear approximation compare to the true function? To do this, generate two grids in Matlab for θ_t and k_t as follows:

$kgrid = [.12 : .001 : .2]$
$thetagrid = [.95 : .01 : 1.2]$
for i=1:length(kgrid)
for j=1:length(thetagrid)
ctrue(i,j)=$(1 - .33 \times .95) \times (kgrid(i)^{.33}) \times thetagrid(j)$;
clinear(i,j)=$B1 + B2 \times kgrid(i) + B3 \times thetagrid(j)$;
end
end

Problem 3. Take the following model with consumption (c_t), labor ((n_t), and capital (k_t). The goal is to maximize the stream of discounted utility of the form:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \tag{2}$$

where the objective is to maximize W s.t. $k_{t+1} = f(k_t, n_t) - c_t$ and $0 \leq n_t \leq 1$.

a. Formulate the Bellman equation for this problem.

b. What do we hope to obtain by solving the above problem? Be specific.

c. Derive the first order conditions and envelope condition.

d. Show that the ratio of the marginal utility of consumption to the marginal utility of leisure depends on the marginal product of labor.

e. Using the envelope condition find the first order conditions absent of the value function.

f. Using the function forms $f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$, $u(c_t, 1 - n_t) = ln(c_t) + ln(1 - n_t)$, show that their exists a unique $n^* = n^{ss}$, and $k^* = k^{ss}$ for which the first-order condition for consumption is satisfied. Assume that $0 < \alpha < 1$.

g. Using the same approach as in Problem 1 part c, solve for steady state values for c_t, n_t, and k_t. You should have 3 equations and 3 unknowns. *Warning: This took about 20 minutes to complete on my computer.*

h. Using the co-state variable $\lambda_t = \beta\lambda_{t+1} f_k(k_{t+1}, n_{t+1})$, linearize your system of equations and the capital constraint.