ECON 6310
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Practice Problems

Problem 1. Take the following population model for a bivarite VAR(2) for the federal funds rate (i_t) and the exchange rate (ex_t):

$$\begin{bmatrix} i_t \\ ex_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \phi_{11}^{(1)} & \phi_{12}^{(1)} \\ \phi_{21}^{(1)} & \phi_{22}^{(1)} \end{bmatrix} \begin{bmatrix} i_{t-1} \\ ex_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_{11}^{(2)} & \phi_{12}^{(2)} \\ \phi_{21}^{(2)} & \phi_{22}^{(2)} \end{bmatrix} \begin{bmatrix} i_{t-2} \\ ex_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix} \quad (1)$$

a. Rewrite the model in vector form using matrices Φ_1 and Φ_2.

b. Form the companion matrix F for the VAR(1) representation.

c. Derive the population impulse response function $\Psi_s(2,1)$ for $s = 0, 1, 2, 3$.

d. Plot by hand, a hypothetical IRF $\Psi_s(2,1)$. What should it look like given what we know about the relationship between interest rates and exchange rates?

Problem 2. Using the *fredget2.m* function, obtain the following data: EXUSEU, EXUSUK, FEDFUNDS, and CPIAUCSL.[1]

a. Explain each variable in detail.

b. Using EXUSEU as the exchange rate, estimate a bivariate VAR(2) with federal funds rate as the first variable with a lag of 2 and horizon 20. Using Matlab, construct $\hat{\Phi}_1$ and $\hat{\Phi}_2$. Verify that your expression obtained in problem 1 part c is correct. [2]

[1] Note that the sample size for each series will be different. When you construct your data vector Y you should make the necessary adjustments.

[2] The default for your inputs in the *vecar.m* file should be as follows: $[PiVar, OmegaP, IRFs, DEC, LagDiag, CI, GC] = vecar(Y, 0, lags, hor, 1, 0, 0, 0, 1, 0, 0)$

c. Find the optimal lag length by minimizing the AIC. To do this place the following code in the command window:

```
lags=1:1:12;
for i=1:12
[PiVar, OmegaP, IRFs, DEC, LagDiag, CI, GC]
= vecar(Y, 0, lags(i), 40, 1, 0, 0, 0, 1, 0, 0);
AIC(i, 1) = LagDiag{3};
end
```

Notice that the AIC is stored for each lag from 1 to 12. At which lag is the AIC minimized?

d. Given the results of Ivanov and Kilian (2005), is the AIC the appropriate statistic to base our lag choice on?

e. At the optimal length, do interest rates Granger cause EXUSEU? Explain what this means in economic terms.

f. What is an estimate of the *correlation* between the errors ϵ_t^1 and ϵ_t^2? To do this you need $\hat{\Omega}$.

g. Using the optimal lag, plot the impulse response function for exchange rates for a shock to the federal funds rate. Be sure to label each axis appropriately. Interpret the results carefully.

h. Now repeat b, c, d, and g using EXUSUK in place of EXUSEU. How do your conclusions change?

i. Finally add log(CPIAUCSL) as the third variable to a VAR(3). Does the inclusion of the CPI change your main results?