ECON 7920
Econometrics II
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Problem Set 3
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Problem 1

Consider the population model that relates a student's term GPA ($termgpa$) to prior to term GPA ($priGPA$) and a student's ACT score (ACT). Assume the functional form for the conditional mean function is given by: $m(x, \theta_0) = \theta_{01} + \theta_{02} priGPA + \theta_{03} ACT$.[1]

a. Modify the qfunction.R file to reflect the population model presented above. Be sure to save the file and then source the file accordingly.

b. Using the functional form above, calculate the analytical expression for the Hessian matrix for each i and each p. Recall that the Hessian matrix should take the following form:

$$H_i = \begin{bmatrix} h_{i,11} & h_{i,12} & h_{i,13} \\ h_{i,21} & h_{i,22} & h_{i,23} \\ h_{i,31} & h_{i,32} & h_{i,33} \end{bmatrix} \tag{1}$$

where $h_{i,11}$ is the derivative of $\frac{\partial q_i}{\partial \theta_{01}}$ with respect to θ_{01}, $h_{i,12}$ is the derivative of $\frac{\partial q_i}{\partial \theta_{01}}$ with respect to θ_{02}, and $h_{i,13}$ is the derivative of $\frac{\partial q_i}{\partial \theta_{01}}$ with respect to θ_{03}.

c. Given your Hessian matrix above, compute the sum of the Hessian:

$$\sum_{i=1}^{N} H_i = \begin{bmatrix} \sum_{i=1}^{N} h_{i,11} & \sum_{i=1}^{N} h_{i,12} & \sum_{i=1}^{N} h_{i,13} \\ \sum_{i=1}^{N} h_{i,21} & \sum_{i=1}^{N} h_{i,22} & \sum_{i=1}^{N} h_{i,23} \\ \sum_{i=1}^{N} h_{i,31} & \sum_{i=1}^{N} h_{i,32} & \sum_{i=1}^{N} h_{i,33} \end{bmatrix} \tag{2}$$

[1] For this problem you will need the script files qfunction.R, qderivfun.R, and qderivfun2.R.

d. Now compute the analytical expression for A_0.

e. Using the *nls* command in R and the dataset *attend.csv*, estimate the population model under consideration.[2]

f. Given your estimates above, verify that the code $qderivfun2.R$ produces the exact values for both $\sum_{i=1}^{N} \ddot{H}_i$ and $\hat{A}_0$.

g. Using your estimates above and the $qderivfun.R$ script, calculate the estimated variance-covariance matrix $\hat{Avar}(\hat{\theta})$.

h. Test the null hypothesis $H_0 : \theta_{0j} = 0$ versus $H_1 : \theta_{0j} \neq 0$ for $j = 1, 2, 3$.

[2] nlsout=nls(termgpa~(b0 + b1*priGPA+b2*ACT), start = list(b0 = 1, b1 = 0.04321,b2=.9))