ECON 7020
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Problem Set 6
Due date: May 6, 2022

Problems from McCandless and Wallace:
Chapter 3 Exercises:
3.3, 3.6, 3.7

Problems from McCandless and Wallace:
Chapter 9 Exercises:
9.1-9.6

Problem 1. Take a two-period OLG model with production. Agents maximize their discounted stream of utility over the two periods of their life. The budget constraint of the young is given by: $c_t^h(t) = w(t)\Delta_t^h(t) - l^h(t) - k^h(t+1)$ and the budget constraint of the old is given by: $c_t^h(t+1) = w(t+1)\Delta_t^h(t+1) + r^l(t)l^h(t) + r^k(t+1)k^h(t+1)$. Suppose utility is given by: $u_t^h = c_t^h(t)[c_t^h(t+1)]^\beta$. Also assume that factor markets are prefectly competitive and the production function is given by $Y(t) = \gamma(t)L(t)^{1-\alpha}K(t)^\alpha$. In addition, let the population be constant so that $N(t) + N(t-1) = 1$ for all t and $\gamma(t+1) = (1+g)\gamma(t)$.

a. Derive the lifetime budget constraint (LBC) and state the no arbitrage condition. Why must the arbitrage condition hold in equilibrium?

b. Derive the individual savings function for an arbitrary agent h.

c. Define a perfect foresight competitive equilibrium.

d. Using the fact that $L(t) = N(t)\Delta_t^h(t) + N(t-1)\Delta_{t-1}^h(t)$ solve for the steady state capital stock.

e. Assuming $g > 0$ find the steady state growth rate of the capital stock. What is the growth rate of output?