ECON 7020
Philip Shaw
Practice Problems

Problems from McCandless and Wallace:
Chapter 9 Exercises:
9.1-9.6

Problem 1. Take a two-period OLG model with production. Agents maximize their discounted stream of utility over the two periods of their life. The budget constraint of the young is given by: $c_t^h(t) = w(t)\Delta_t^h(t) - l^h(t) - k^h(t+1)$ and the budget constraint of the old is given by: $c_t^h(t+1) = w(t+1)\Delta_t^h(t+1) + r^l(t)l^h(t) + r^k(t+1)k^h(t+1)$. Suppose utility is given by $u_t^h = c_t^h(t)[c_t^h(t+1)]^\beta$. Also assume that factor markets are perfectly competitive and the production function is given by $Y(t) = \gamma(t)L(t)^{1-\alpha}K(t)^\alpha$. In addition, let the population be constant $N(t) + N(t-1) = 1$ for all t and $\gamma(t+1) = (1+g)\gamma(t)$.

a. Derive the lifetime budget constraint (LBC) and state the no arbitrage condition. Why must the arbitrage condition hold in equilibrium?

b. Derive the individual savings function for an arbitrary agent h.

c. Define a perfect foresight competitive equilibrium.

d. Using the fact that $L(t) = N(t)\Delta_t^h(t) + N(t-1)\Delta_{t-1}^h(t)$ solve for the steady state capital stock.

e. Assuming $g > 0$ find the steady state growth rate of the capital stock. What is the growth rate of output?

Problem 2. Take the Ayagari (1994) model as discussed in class.

a. How does he motivate the use of heterogenous agent models as an alternative to representative agent models?

b. Explain figures Ia and Ib in detail.

c. Describe his solution method in detail. How does it differ from the approach discussed in class?

d. What are the main results of the paper?

Problem 3. Consider the dynamic programming problem in Hansen and Imrohoroğlu (1992):

$$V(m,s,\eta) = \begin{cases} \max\limits_{m'}\Big\{U(m+(1-\tau)\theta y - m', 1) + \beta \sum_{s'} \chi(u,s')V(m',s',0)\Big\}, & s=u \\ \max\Big\{\max\limits_{m'}\{U(m+(1-\tau)y - m', 1-\hat{h}) + \beta \sum_{s'} \chi(e,s')V(m',s',1)\Big\}, & \\ \pi(\eta)\Big[\max\limits_{m'}\Big\{U(m+(1-\tau)\theta y - m', 1) + \beta \sum_{s'} \chi(e,s')V(m',s',0)\Big\}\Big] & \\ [1-\pi(\eta)]\Big[\max\limits_{m'}\Big\{U(m - m', 1) + \beta \sum_{s'} \chi(e,s')V(m',s',0)\Big\}\Big]\Big\}, & s=e \end{cases}$$

subject to $m' \geq 0$.

a. Explain, in detail, the Bellman equation above.

b. Define a stationary competitive equilibrium.

c. Describe how Hansen and Imrohoroğlu (1992) solve for a stationary competitive equilibrium.

d. Graph the policy function $m'(x)$ for different levels of current assets m across different values of the state vector x.

e. What are the main results of their paper.

f. Modify the Bellman equation to allow for continuous labor choice. How would this alter the main results of the paper?