

ECON 7020
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Problem Set 5
Due date: April 30, 2025

Problems from McCandless and Wallace:

Chapter 9 Exercises:

9.1-9.6

Problem 1. Take a simple two-period heterogeneous agent OLG model. Suppose we have agents that differ in their abilities a_i for $i = l, h$. Low ability agents are assigned the value $a_l = 1$ and high ability agents are assigned the value $a_h = 2$. The population of high ability agents is given by N^h and the population of low ability agents is given by N^l . Assume that both types of agents have the same utility function $u_t^h = c_t^h(t)c_t^h(t+1)$ and that the population of each type is constant over time. Furthermore assume that the ability level of each agents allows them to transform their endowments when young such that $w_t^h = [a_i\tilde{w}_t^h(t), \tilde{w}_t^h(t+1)]$ for $i = l, h$. Assume that each type of agent is assigned a pre-transformed endowment of $\tilde{w}_t^h = [1, 1]$. Assume that generations transition over time in ability according to the following transition matrix:

$$P = \begin{bmatrix} p_{hh} & p_{hl} \\ p_{lh} & p_{ll} \end{bmatrix} \quad (1)$$

where p_{hh} gives the probability of high ability agents giving “birth” to high ability agents and p_{hl} is the probability of high ability agents giving “birth” to low ability agents. We can think of ability as following a Markov chain (a, P, π_0) where a is the ability type, P is a transition matrix, and π_0 is the initial distribution of each type of agent. Assume that $N^h + N^l = 1$ where the population of agents at time t is given by the proportion of each type of agent contained in $\pi_t = \begin{bmatrix} .9 \\ .1 \end{bmatrix}$. Furthermore assume that $p_{ij} = .5$ for all i and j .

- a. How does the stochastic nature of the model impact the decision of each individual?
- b. What is the time t equilibrium interest rate?

- c. What is the stationary distribution for each type of agent?
- d. Define a stationary competitive equilibrium.
- e. Solve for the stationary equilibrium interest rate. How does this interest rate compare to the one computed in part b? What are the individual savings for each type of consumer? What are the individual consumption levels for each type of consumer?

Problem 2. Take the Aiyagari (1994) model as discussed in class.

- a. How does he motivate the use of heterogeneous agent models as an alternative to representative agent models?
- b. Explain figures Ia and Ib in detail.
- c. Describe his solution method in detail.
- d. What are the main results of the paper?