ECON 7020
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Practice Problems

Problem 1.
a. Start with the consumption Euler equation. Re-write the Euler equation in terms of the stochastic discount factor, pointing out what the stochastic discount factor is.

b. Derive the fundamental result of the C-CAPM model using the fact that: $E_t[X_{t+1}Y_{t+1}] = E_t[X_{t+1}]E_t[Y_{t+1}] + cov_t(X_{t+1}, Y_{t+1})$. The fundamental result of the C-CAPM model shows the relationship between the expected return on the risky asset in excess of the safe asset. Explain briefly.

c. Use these things to explain the equity premium puzzle.

d. Describe (in words and mathematically) how the equity premium puzzle could be resolved.

Problems from McCandless and Wallace:
Chapter 1 Exercises:
1.1-1.4, 1.11

Chapter 2 Exercises:
2.1, 2.5

Chapter 3 Exercises:
3.2

Problem 2. Take a simple two-period heterogeneous agent OLG model. Suppose we have agents that differ in their abilities a_i for $i = l, h$. Low ability agents are assigned the value $a_l = 1$ and high ability agents are assigned the value $a_h = 2$. The population of high ability agents is given by N^h and the population of low ability agents is given by N^l. Assume that both types of agents have the same utility function $u_t^h = c_t^h(t)c_t^h(t+1)$ and that the population of each type is constant over time. Furthermore assume that the ability level of each agents allows them to transform their endowments when young such that $w_t^h = [a_i\tilde{w}_t^h(t), \tilde{w}_t^h(t+1)]$ for $i = l, h$. Assume that each type of agent is assigned a pre-transformed endowment of $\tilde{w}_t^h = [1, 1]$.

a. Define a competitive equilibrium.

b. Assuming $N^h = 100$ and $N^l = 50$, solve the the competitive equilibrium. What is the equilibrium interest rate? What the the savings of the high ability and low ability agents? What are the consumption levels for each type of agent?

c. Describe in words the trading arrangements between the high and low ability agents. Do they make sense?

d. Now suppose we fix $N^l = 50$ but allow the population of high ability agents to remain unspecified at N^h. What is the limiting behavior of the interest rate as population of high ability agents becomes arbitrarily large or small? What is the limiting behavior of the individual savings functions? Explain your results intuitively. (Note: You should be able to say exactly what the limiting behavior is.)

Problem 3. Building on the model presented in the first problem, assume that generations transition over time in ability according to the following transition matrix:

$$P = \begin{bmatrix} p_{hh} & p_{hl} \\ p_{ll} & p_{lh} \end{bmatrix} \tag{1}$$

where p_{hh} gives the probability of high ability agents giving "birth" to high ability agents and p_{hl} is the probability of high ability agents giving "birth" to low ability agents. We can think of ability as following a Markov chain (a, P, π_0) where a is the ability type, P is a transition matrix, and π_0 is the initial distribution of each type of agent. Assume that $N^h + N^l = 1$ where the population of agents at time t is given by the proportion of each type of agent contained in π_0. Furthermore assume that $p_{ii} = .5$ for all i.

a. How does the stochastic nature of the model impact the decision of each individual?

b. Define a competitive equilibrium.

c. Solve for the equilibrium interest rate. What is the individual savings for each type of consumer? What proportion of agents are high ability and low ability in equilibrium?