ECON 7020
Philip Shaw
Problem Set 4
Due Date: April 24, 2024

**Problems from McCandless and Wallace:**
Chapter 1 Exercises:
1.1-1.4, 1.11

Chapter 2 Exercises:
2.1, 2.5

**Problem 1**. Take a simple two-period heterogeneous agent OLG model. Suppose we have agents that differ in their abilities $a_i$ for $i = l, h$. Low ability agents are assigned the value $a_l = 1$ and high ability agents are assigned the value $a_h = 2$. The population of high ability agents is given by $N^h$ and the population of low ability agents is given by $N^l$. Assume that both types of agents have the same utility function $u_t^h = c_t^h(t)c_t^h(t+1)$ and that the population of each type is constant over time. Furthermore assume that the ability level of each agents allows them to transform their endowments when young such that $w_t^h = [a_i\tilde{w}_t^h(t), \tilde{w}_t^h(t+1)]$ for $i = l, h$. Assume that each type of agent is assigned a pre-transformed endowment of $\tilde{w}_t^h = [1, 1]$.

a. Define a competitive equilibrium.

b. Assuming $N^h = 100$ and $N^l = 50$, solve the the competitive equilibrium. What is the equilibrium interest rate? What the the savings of the high ability and low ability agents? What are the consumption levels for each type of agent?

c. Describe in words the trading arrangements between the high and low ability agents. Do they make sense?

d. Now suppose we fix $N^l = 50$ but allow the population of high ability agents to remain unspecified at $N^h$. What is the limiting behavior of the interest rate as population of high ability agents becomes arbitrarily large or small? What is the limiting behavior of the individual savings functions? Explain your results intuitively. (Note: You should be able to say exactly what the limiting behavior is.)