

ECON 7020
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Problem Set 4
Due date: April 20, 2023

Problems from McCandless and Wallace:

Chapter 9 Exercises:

9.1-9.6

Problem 1. Take a two-period OLG model with production. Agents maximize their discounted stream of utility over the two periods of their life. The budget constraint of the young is given by: $c_t^h(t) = w(t)\Delta_t^h(t) - l^h(t) - k^h(t+1)$ and the budget constraint of the old is given by: $c_t^h(t+1) = w(t+1)\Delta_t^h(t+1) + r^l(t)l^h(t) + r^k(t+1)k^h(t+1)$. Suppose utility is given by: $u_t^h = c_t^h(t)[c_t^h(t+1)]^\beta$. Also assume that factor markets are perfectly competitive and the production function is given by $Y(t) = \gamma(t)L(t)^{1-\alpha}K(t)^\alpha$. In addition, let the population be constant so that $N(t) + N(t-1) = 1$ for all t and $\gamma(t+1) = (1+g)\gamma(t)$.

- a. Derive the lifetime budget constraint (LBC) and state the no arbitrage condition. Why must the arbitrage condition hold in equilibrium?
- b. Derive the individual savings function for an arbitrary agent h .
- c. Define a perfect foresight competitive equilibrium.
- d. Using the fact that $L(t) = N(t)\Delta_t^h(t) + N(t-1)\Delta_{t-1}^h(t)$ solve for the steady state capital stock.
- e. Assuming $g > 0$ find the steady state growth rate of the capital stock. What is the growth rate of output?