ECON 7020
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Problem Set 4
Due date: April 8, 2022

Problem 1. Suppose we have an economy with $j = 1, ..., n$ risky assets A^j_{t+i+1} all with their own rate of return r^j_{t+i+1}. The dynamic budget constraint is given by $\sum_{j=1}^n A^j_{t+i+1} = \sum_{j=1}^n (1 + r^j_{t+i})A^j_{t+i} + y_{t+1} - c_{t+i}$. Assume utility is defined by $u(c_t)$ and satisfies the assumptions required for risk aversion.

a. Formulate the Bellman equation carefully pointing out the state(s) and controls.

b. Prove that the first order condition for each asset j is given by:

$$u'(c_t) = \frac{1}{1+\rho} E_t[(1 + r^j_{t+1})u'(c_{t+1})] \tag{1}$$

c. Re-write the Euler equation in terms of the stochastic discount factor, pointing out what the stochastic discount factor represents.

d. Derive the fundamental result of the C-CAPM model using the fact that : $E_t[X_{t+1}Y_{t+1}] = E_t[X_{t+1}]E_t[Y_{t+1}] + cov_t(X_{t+1}, Y_{t+1})$. The fundamental result of the C-CAPM model shows the relationship between the expected return on the risky asset in excess of the safe asset.

e. Now assume utility takes the following functional form $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$. Find the expression for the stochastic discount factor and explain how the excess return depends on the covariance between the stochastic discount factor and a risky asset's return.

Problems from McCandless and Wallace:
Chapter 1 Exercises:
1.1-1.4, 1.11