

ECON 7020
Philip Shaw
Practice Problems

Problem 1. Suppose we have an economy with $j = 1, \dots, n$ risky assets A_{t+i+1}^j all with their own rate of return r_{t+i+1}^j . The dynamic budget constraint is given by $\sum_{j=1}^n A_{t+i+1}^j = \sum_{j=1}^n (1 + r_{t+i}^j) A_{t+i}^j + y_{t+i} - c_{t+i}$. Assume utility is defined by $u(c_t)$ and satisfies the assumptions required for risk aversion.

a. Formulate the Bellman equation carefully pointing out the state(s) and controls.

b. Prove that the first order condition for each asset j is given by:

$$u'(c_t) = \frac{1}{1 + \rho} E_t[(1 + r_{t+1}^j)u'(c_{t+1})] \quad (1)$$

c. Re-write the Euler equation in terms of the stochastic discount factor, pointing out what the stochastic discount factor is.

d. Derive the fundamental result of the C-CAPM model using the fact that: $E_t[X_{t+1}Y_{t+1}] = E_t[X_{t+1}]E_t[Y_{t+1}] + cov_t(X_{t+1}, Y_{t+1})$. The fundamental result of the C-CAPM model shows the relationship between the expected return on the risky asset in excess of the safe asset.

e. Now assume that utility takes the following functional form $u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma}$. Find the expression for the stochastic discount factor and explain how the excess return depends on the covariance between the stochastic discount factor and a risky asset's return.

Problem 2. Suppose the consumer maximizes the following objective function:

$$\max E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) \quad (2)$$

subject to the dynamic budget constraint:

$$A_{t+i+1} = (1+r)[A_{t+i} + y_{t+i} - c_{t+i}] \quad (3)$$

where

$$y_{t+1} = y_t + \epsilon_{t+1} \quad (4)$$

and $\epsilon_{t+1} \sim N(0, \sigma^2)$

- a. Under what circumstances do we get a “certainty equivalent result”?
- b. Now assume that the utility function is of the exponential form, e.g., $u(c_t) = -(\frac{1}{\alpha})e^{-\alpha c_t}$ where $\alpha > 0$. Calculate the measure of relative risk aversion.
- c. For a general utility function $u(c_t)$, *derive* the coefficient of absolute prudence. What is the coefficient of absolute prudence for the utility function mentioned above?
- d. How does the existence of prudent behavior alter the optimal consumption path found under the certainty equivalent result?