ECON 7020
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Problem Set 3
Due date: March 6, 2024

Problem 1. Assume a quadratic utility, rational expectations framework and assume that the rate of time preference, ρ equals the interest rate, r. Assume that labour income follows the following stochastic process:

$$y_{t+1} = \lambda y_t + (1 - \lambda)\bar{y} + \epsilon_{t+1} \tag{1}$$

where $E_t \epsilon_{t+1} = 0$ and ϵ_{t+1} is an income innovation, $0 \geq \lambda \leq 1$and $\bar{y}$ is the unconditional mean of labour income.

1. Prove that the consumption function in this case has the following form:

$$c_t = rA_t + \frac{r}{1 + r - \lambda} y_t + \frac{1 - \lambda}{1 + r - \lambda} \bar{y}. \tag{2}$$

2. What happens if $\lambda = 1$? Explain.
3. What happens if $\lambda = 0$? Explain.

Problem 2. Suppose a consumer maximizes the following objective function:

$$max E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) \tag{3}$$

subject to the dynamic budget constraint:

$$A_{t+i+1} = (1+r)[A_{t+i} + y_{t+i} - c_{t+i}] \tag{4}$$

where

$$y_{t+1} = y_t + \epsilon_{t+1} \tag{5}$$

and $\epsilon_{t+1} \sim N(0, \sigma^2)$.

1. Under what circumstances do we get a "certainty equivalent result"?

2. Now assume that the utility function is of the exponential form, e.g., $u(c_t) = -(\frac{1}{\alpha})e^{-\alpha c_t}$ where $\alpha > 0$. Calculate the measure of relative risk aversion.

3. For a general utility function $u(c_t)$, *derive* the coefficient of absolute prudence. What is the coefficient of absolute prudence for the utility function mentioned above?

4. How does the existence of prudent behavior alter the optimal consumption path found under the certainty equivalent result?