ECON 7020
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Problem Set 3
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Problem 1. Suppose we have an economy with $j = 1, ..., n$ risky assets A^j_{t+i+1} all with their own rate of return r^j_{t+i+1}. The dynamic budget constraint is given by $\sum_{j=1}^n A^j_{t+i+1} = \sum_{j=1}^n (1 + r^j_{t+i})A^j_{t+i} + y_{t+1} - c_{t+i}$. Assume utility is defined by $u(c_t)$ and satisfies the assumptions required for risk aversion.

a. Formulate the Bellman equation carefully pointing out the state(s) and controls.

b. Prove that the first order condition for each asset j is given by:

$$u'(c_t) = \frac{1}{1+\rho} E_t[(1 + r^j_{t+1})u'(c_{t+1})] \tag{1}$$

c. Re-write the Euler equation in terms of the stochastic discount factor, pointing out what the stochastic discount factor represents.

d. Derive the fundamental result of the C-CAPM model using the fact that : $E_t[X_{t+1}Y_{t+1}] = E_t[X_{t+1}]E_t[Y_{t+1}] + cov_t(X_{t+1}, Y_{t+1})$. The fundamental result of the C-CAPM model shows the relationship between the expected return on the risky asset in excess of the safe asset.

e. Now assume utility takes the following functional form $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$. Find the expression for the stochastic discount factor and explain how the excess return depends on the covariance between the stochastic discount factor and a risky asset's return.

Problem 2. Suppose a consumer maximizes the following objective function:

$$max E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}) \tag{2}$$

subject to the dynamic budget constraint:

$$A_{t+i+1} = (1+r)[A_{t+i} + y_{t+i} - c_{t+i}] \tag{3}$$

where

$$y_{t+1} = y_t + \epsilon_{t+1} \tag{4}$$

and $\epsilon_{t+1} \sim N(0, \sigma^2)$.

a. Under what circumstances do we get a "certainty equivalent result"?

b. Now assume that the utility function is of the exponential form, e.g., $u(c_t) = -(\frac{1}{\alpha})e^{-\alpha c_t}$ where $\alpha > 0$. Calculate the measure of relative risk aversion.

c. For a general utility function $u(c_t)$, *derive* the coefficient of absolute prudence. What is the coefficient of absolute prudence for the utility function mentioned above?

d. How does the existence of prudent behavior alter the optimal consumption path found under the certainty equivalent result?