

ECON 7020  
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Problem Set 1  
Due date: Feb. 14, 2023

**Problem 1.** Take the generalized model for stochastic dynamic programming. Let  $r(x, u)$  be the return function and  $x' = g(x, u, z, \phi)$  be the transition function. Under this framework assume we have two shocks to the model  $z$  and  $\phi$ .

- a. Describe the transition function for the exogenous states  $z$  and  $\phi$ .
- b. Formulate the Bellman equation explicitly using the transition function for the exogenous shocks  $z$  and  $\phi$ .
- c. Find the first order condition for the control vector  $u$  and the Benveniste-Sheinkman condition.
- d. What conditions would one have to place on the stochastic processes  $z$  and  $\phi$  so that usual contraction mapping theorem still applies?

**Problem 2.** Start with the stochastic growth model for which  $u(c_t) = \ln(c_t)$ ,  $k_{t+1} = z_t k_t^\alpha - c_t$  where we set  $A = 1$  and  $\delta = 1$ . Furthermore assume shocks to productivity take only two values defined by:

$$z_t = \begin{cases} z^H, & \text{w.p. } p^H \\ z^L, & \text{w.p. } p^L = (1 - p^H) \end{cases}$$

where  $z^H > z^L$  and  $z_{t+1} \perp z_t$ .

- a. Formulate the Bellman equation using the conditional expectations operator.
- b. What explicit form does the conditional expectations function take in the Bellman equation formed above?
- c. Perform four iterations on the Bellman equation. What should your initial guess be for the value function? Does this initial guess matter?

**Problem 3.** Take the following model with consumption  $(c_t)$ , labor  $((n_t))$ , and capital  $(k_t)$ . The goal is to maximize the stream of discounted utility of the form:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \quad (1)$$

where the objective is to maximize  $W$  s.t.  $k_{t+1} = f(k_t, n_t) - c_t$  and  $0 \leq n_t \leq 1$ .

- a. Formulate the Bellman equation for this problem.
- b. What do we hope to obtain by solving the above problem? Be specific.
- c. Derive the first order conditions and envelope condition.
- d. Show that the ratio of the marginal utility of consumption to the marginal utility of leisure depends on the marginal product of labor.
- e. Using the envelope condition find the first order conditions absent of the value function.