ECON 7020
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Problem Set 1
Due date: Feb. 7, 2024

Problem 1. Take the generalized model for stochastic dynamic programming. Let $r(x, u)$ be the return function and $x' = g(x, u, z, \phi)$ be the transition function. Under this framework assume we have two shocks to the model z and ϕ.

a. Describe the transition function for the exogenous states z and ϕ.

b. Formulate the Bellman equation explicitly using the transition function for the exogenous shocks z and ϕ.

c. Find the first order condition for the control vector u and the Benveniste-Sheinkman condition.

d. What conditions would one have to place on the stochastic processes z and ϕ so that usual contraction mapping theorem still applies?

Problem 2. Start with the stochastic growth model for which $u(c_t) = ln(c_t)$, $k_{t+1} = z_t k_t^\alpha - c_t$ where we set $A = 1$ and $\delta = 1$. Furthermore assume shocks to productivity take only two values defined by:

$$z_t = \begin{cases} z^H, & \text{w.p. } \; p^H \\ z^L, & \text{w.p. } \; p^L = (1 - p^H) \end{cases}$$

where $z^H > z^L$ and $z_{t+1} \perp\!\!\!\perp z_t$.

a. Formulate the Bellman equation using the conditional expectations operator.

b. What explicit form does the conditional expectations function take in the Bellman equation formed above?

c. Perform four iterations on the Bellman equation. What should your initial guess be for the value function? Does this initial guess matter?

Problem 3. Take the following model with consumption (c_t), labor ((n_t), and capital (k_t). The goal is to maximize the stream of discounted utility of the form:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \tag{1}$$

where the objective is to maximize W s.t. $k_{t+1} = f(k_t, n_t) - c_t$ and $0 \leq n_t \leq 1$.

a. Formulate the Bellman equation for this problem.

b. What do we hope to obtain by solving the above problem? Be specific.

c. Derive the first order conditions and envelope condition.

d. Show that the ratio of the marginal utility of consumption to the marginal utility of leisure depends on the marginal product of labor.

e. Using the envelope condition find the first order conditions absent of the value function.