ECON 5760
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Problem Set 5

Problem 1. Consider the benchmark model with shocks to output with $u(C_t) = \frac{C_t^{1-\eta}-1}{1-\eta}$ and $F(Z_t, K_t) = Z_t K_t^{\alpha}$ with the resource constraint $f(K_t) = C_t + K_{t+1}$ where $f(K_t) = F(Z_t, K_t) + (1-\delta)K_t$.

a. Formulate the Bellman equation for the problem and derive the euler equation.

b. Using the euler equation at the steady-state level consumption (C^*) and capital stock (K^*), solve for the steady-state capital stock for an arbitrary set of parameters η, δ, α, β, and Z_j.

c. Write a function called simplevalueitstoch.m to solve for the policy function on a grid of size $n = 50$ and $m = 9$ by simple value function iteration. As a shortcut, you can start with the code simplevalueit.m and modify it to reflect the addition of shocks to output. How long does it take to converge to the terminal solution?[1]

d. We know that for the case in which $\delta = 1$ and $\eta = 1$ that the policy function for capital is given by $h(K, Z) = \alpha\beta Z K^{\alpha}$. Plot the approximate solution $\hat{h}(K, Z)$ against the true solution $h(Z, K)$. Calculate $max|\hat{h}(K, Z) - h(Z, K)|$.

e. Now save the terminal value for the value function v^* as $v0$. Define a new grid of capital stock of size $n = 500$ with the same lower bound and upper bound for capital stock as before. Using the old grid for capital and $v0$, interpolate the value of $v0$ onto the new grid for capital stock of size $n = 500$. How long does value function iteration take to converge with this interpolated value for $v0$? Now graph the approximate solution $\hat{h}(K, Z)$ against the true solution $h(Z, K)$. Calculate $max|\hat{h}(K, Z) - h(Z, K)|$. How does this compare to what you got in part d?

f. Write a program that is capable of evaluating the RHS of the Euler equation for a value ϵ as presented in class:

[1] Use the code tauchenAR.m to generate the discrete approximation for Z_j with $\rho = .95$, $\sigma = .01$, and $\lambda = 3$.

$$\phi(K,Z,\sigma,\epsilon) = \beta[\hat{h}^C(\hat{h}^K(K,Z,\sigma), e^{\rho Z_t+\sigma\epsilon}, \sigma)]^{-\eta}[1-\delta+\alpha e^{\rho Z_t+\sigma\epsilon}(\hat{h}^K(K,Z,\sigma))^{\alpha-1}]$$

Note that to do this you must be able to interpolate the policy function $\hat{h}^K(K,Z,\sigma)$ to off-grid values for Z and K. To accomplish this, you can use bilinear interpolation. Run the code simplevalueitstoch.m which will provide the policy function $\hat{h}^K(K,Z,\sigma)$. Also notice that you should increase $\lambda = 10$ (for the AR(1) discrete approximation) before solving for the optimal policy function.