ECON 5760
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Problem Set 5

Problem 1. Consider the benchmark model with $u(C_t) = \frac{C_t^{1-\eta}-1}{1-\eta}$ and $F(K_t) = K_t^{\alpha}$ with the resource constraint $f(K_t) = C_t + K_{t+1}$ where $f(K_t) = F(K_t) + (1-\delta)K_t$.

a. Formulate the Bellman equation for the problem and derive the euler equation.

b. Using the euler equation at the steady-state level consumption (C^*) and capital stock (K^*), solve for the steady-state capital stock for an arbitrary set of parameters η, δ, α, and β.

c. Write a function called utility.m that defines the utility for a vector of consumptions C_t and a value for η. Note that you should include an if statement for the utility function when $\eta = 1$ in which $u(C_t) = ln(C_t)$.

d. Using the simple value function algorithm discussed in class, write a code call simplevalueit.m that solves for the optimal policy function for $K_{t+1} = h(K_t)$ for assigned parameter values $\alpha = .27$, $\beta = .994$, $\eta = 2$, $\epsilon = .01$, and $\delta = .011$. The capital stock should be defined on the grid $[.75K^*, 1.25K^*]$. Note that when you define consumption in the model is can be negative for certain values of capital across the grid of values. To prevent negative consumption, set consumption equal to $C_t = \epsilon_2$ when consumption takes negative values where $\epsilon_2 = .000001$. Finally, your initial guess for the value function v_i^0 should be a $n \times 1$ vector of zeros where $n = 250$.

e. How long does your code take to complete value function iteration? Now increase the grid size to $n = 500$. Now how long does your code take to complete?

f. Now plot your policy function $K' = h(K)$ against K. Show that if $K < K^*$ then the optimal $K' > K$ and when $K > K^*$ then the optimal $K' < K$.

g. Starting from an intial capital stock $K_0 = .85K^*$, simulate a time path for capital stock for $T = 250$. Does capital stock converge to a steady state

level capital stock? How does this value compare to K^* from the analytical solution?

h. Using the relationship $C_t = K_t^\alpha + (1-\delta)K_t - K_{t+1}$, calculate the optimal time path for consumption $C_t = \phi(K_t)$ for the same sequence of capital stocks in part g.