ECON 5760
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Problem Set 3

Problem 1. For this problem write a script file called NC.m that implements the Newton-Cotes method of integration for an arbitrary function $f(x)$. It should take as inputs the function and the limits of integration $[a, b]$ and output the value of the definite integral. Specifically, you should use the Trapezoid rule as presented in Equation (11.73)

a. Using your code above, integrate over the support $x \in [-3, 0]$ for $\phi(x)$ where $\phi(x)$ is the standard normal p.d.f.. How accurate is this approximation?

b. It is well known that the Trapezoid rule gives a more accurate approximation if the intervals are broken-up into smaller intervals so that: $I_1 = [a, b_1]$, $I_2 = [b_1, b_2]$, $I_3 = [b_2, b_3]$,...,$I_{n-1} = [b_{n-1}, b_n]$ where $b_n = b$. Write a program that implements this strategy using your NC.m code from above. It should be able to complete the task for an arbitrary n. How many sub-intervals must be created to get an "accurate" approximation of $\int_{-3}^{0} \phi(x)$?

Problem 2. For this problem you are to write a script (m-file) called NR.m that is capable of implementing the Newton-Raphson method for solving a general system of equations for which $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. This function should be able to take as an input a general system of equations and an initial guess of $\mathbf{x_0}$.

a. Using your function above, find the solution to the following system of equations for an arbitrary q:

$$\begin{aligned}
\frac{1}{k_t^{\alpha} - k_{t+1}} &= \frac{\alpha\beta k_{t+1}^{\alpha-1}}{k_{t+1}^{\alpha} - k_{t+2}} \\
\frac{1}{k_{t+1}^{\alpha} - k_{t+2}} &= \frac{\alpha\beta k_{t+2}^{\alpha-1}}{k_{t+2}^{\alpha} - k_{t+3}} \\
. &= . \\
. &= . \\
. &= . \\
\frac{1}{k_{t+q-2}^{\alpha} - k_{t+q-1}} &= \frac{\alpha\beta k_{t+q-1}^{\alpha-1}}{k_{t+q-1}^{\alpha} - k_{t+q}}
\end{aligned}$$