ECON 5760
Philip Shaw
Problem Set 2

Problem 1. Write a function called lip.m that approximates a function $f(x)$ using linear interpolation. The inputs should be two values x_1 and x_2 along with the two corresponding values $f(x_1)$ and $f(x_2)$ for which the function is defined. In addition to this, the function should also take as an input the value of x which the function is to be approximated at and the output should be the approximate value for $f(x)$, $\hat{f}(x)$.

a. Use this code to estimate the value of $f(x) = ln(x)$ at the point $x = 1$ with the neighboring points being $x_1 = .80$ and $x_2 = 1.2$. How close is the approximate value to the true value? What happens to the quality of the approximation as you choose points closer to the point of approximation $x = 1$?

b. We often want to evalute the function on a range of values for x instead of a single point. Write a vectorized version of your code called liploop.m that does this using a loop over the range of values for x.

c. Now write a function accomplishes the same task as in part b but does not use loops. Call this file lipvec.m.

Problem 2. For this problem you are to write a script (m-file) called bilinear.m that is capable of implementing bilinear interpolation for an arbitrary function $f(x_1, x_2)$.

a. Using your function above, approximate the function $f(x_1, x_2) = ln(x_1) + ln(x_2)$ at points $x_1 = 2$ and $x_2 = 3$ using the points $x_{1i} = 1.90$, $x_{1i+1} = 2.10$, $x_{1j} = 2.90$, $x_{1j+1} = 3.10$ to do a bilinear approximation. What is the absolute deviation between $\hat{f}(x_1, x_2)$ and $f(x_1, x_2)$ at the points under consideration?

b. Now consider the function below:

$$f(x_1, x_2) = \begin{cases} 4x_1 + 3x_2, & \text{if } x_1, x_2 \leq 2 \\ x_1^2 + x_2^2, & \text{if } x_1, x_2 > 2 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Using the surf.m function in Matlab, graph the function over the range of values $x_1 \in (1, 4)$ and $x_2 \in (1, 4)$. Label the graph appropriately.

Using your bilinear.m code, approximate the function above at the points $x_1 = 2.1$ and $x_2 = 2.05$. How good is the linear approximation?

Problem 3. Write a function called CDJac.m that calculates the Jacobian for an arbitrary multivariate function $f(x_1, x_2, ...x_k)$ at any point in the support of $x_1, x_2, ...x_k$.

a. Use this code to approximate the derivative of the multivariate function $f(x_1, x_2) = ln(x_1) + ln(x_2)$. What is the approximation error of your estimate?

b. Now, use your code to estimate the Jacobian of the function defined by Equation (1) above. Is the derivative accurate across the entire support for x_1 and x_2?